Statistical Methods for Data Science

**Mini Project #1**

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Given:

fT(t) = 0.2 exp(−0.1t) − 0.2 exp(−0.2t), 0 ≤ t < ∞,

0, otherwise,

and E(T) = 15 years.

(b)

The figure below represents the histogram of 1000 draws of ‘T’ super-imposed with PDF fT(t). I have observed the similarities between the histogram and density as expected. The seven Monte Carlo estimates for E(T) and P(T > 15) based on 10000 draws and the corresponding solutions are shown in the table below. 1000 and 100000 based estimates are also represented in the table.

Table:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  | Replication Number | | | | | | |
| Parameter | True value | # Draws | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| E(T) | 15 | 1000 | 15.884 | 14.764 | 15.146 | 15.157 | 15.250 | 15.492 | 14.963 |
| E(T) | 15 | 10000 | 14.981 | 15.001 | 15.018 | 15.103 | 14.875 | 15.100 | 15.151 |
| E(T) | 15 | 100000 | 15.016 | 14.969 | 15.049 | 15.016 | 15.017 | 14.990 | 15.003 |
| P(T>15) | 0.396473 | 1000 | 0.426 | 0.384 | 0.397 | 0.402 | 0.397 | 0.429 | 0.402 |
| P(T>15) | 0.396473 | 10000 | 0.396 | 0.395 | 0.399 | 0.402 | 0.392 | 0.400 | 0.404 |
| P(T>15) | 0.396473 | 100000 | 0.398 | 0.395 | 0.399 | 0.396 | 0.397 | 0.397 | 0.397 |

Figure:



R Program:

> set.seed(243)

> x <- replicate(10000, max(rexp(1, (1/10)), rexp(1, (1/10))))

> hist(x, prob = T, ylim = c(0, 0.1), xlab = "time", ylab = "Probability", main = "Histogram

")

> pdf.f <- function(x){

+ return(0.2\*exp(-0.1\*x) - 0.2\*exp(-0.2\*x))

+ }

> curve(pdf.f, from = 0, to = max(x), add = T)

> simulation.f <- function(nsim, lambda.A = (1/10), lambda.B = (1/10)){

+ x <- replicate(nsim, max(rexp(1, lambda.A), rexp(1, lambda.B)))

+ result <- c(mean = mean(x), prob = mean(x > 15))

+ return(result)

+ }

> set.seed(243)

>

> round(replicate(7, simulation.f(1000)), 3)

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

mean 15.884 14.764 15.146 15.157 15.250 15.492 14.963

prob 0.426 0.384 0.397 0.402 0.397 0.429 0.402

> round(replicate(7, simulation.f(10000)), 3)

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

mean 14.981 15.001 15.018 15.103 14.875 15.1 15.151

prob 0.396 0.395 0.399 0.402 0.392 0.4 0.404

> round(replicate(7, simulation.f(100000)), 3)

[,1] [,2] [,3] [,4] [,5] [,6] [,7]

mean 15.016 14.969 15.049 15.016 15.017 14.990 15.003

prob 0.398 0.395 0.399 0.396 0.397 0.397 0.397

(c)

From LLN, approximations are close to true value. It is expected that the approximation accuracy improves as the # of draws increases. We repeat the process until a desired approximation is observed. After certain draws the approximations stall and will not improve the accuracy. As we can observe in the table, the results are consistent. The five approximations of E(T) and P(T>15) are almost identical to the true value. Variability in the approximation is decreased from true value when draws are increased from 1000 to 10000. The increase in approximation accuracy is less noticeable if the draws are increased from 10000 to 100000.

2.

In the hint it was given that unit square is inscribed in the circle. So A be the event which selects a random point of the square is inside the circle.

P(A) = (area of circle)/ (area of square) = Pi/4.

i.e., Pi = 4P(A). we can use the approximate of p(A) using Monte Carlo which is multiplied by 4 gives the approximate value of Pi. It generates a random point in the square. By simulating x and y coordinates of the point as an independent draw from a Uniform (0,1) distribution. The point (x,y) will be in the circle if its less than or equal to the radius of circle.

sqrt{(x-0.5)2 + (y-0.5)2} <= 0.5

to approximate the P(A), we need to simulate 10000 points in the square and apply the above equation to find if the point is inside the circle. Using R, P(A) is approximated to 0.7894.

Pi = 4 x (0.7894) = 3.1576

The above value when compared to 3.1416, which is the true value of Pi, it is approximately equal. With increase in the simulation to a greater value can increase the accuracy of the approximation.

R Program:

> pi.fun <- function(nsim){

+ x <- runif(nsim)

+ y <- runif(nsim)

+ prob <- mean(sqrt((x-0.5)^2 + (y-0.5)^2) <= 0.5)

+ pi <- 4\*prob

+ return(c(prob = prob, pi = pi))

+ }

> set.seed(123)

> pi.fun(10000)

prob pi

0.7894 3.1576

> pi

[1] 3.141593